I. Terminal Value Calculation

- Perpetuity Cash Flow – either growing / stable → use \( \frac{CF_{t+1}}{(r-g)} \) ~ (Gordon Growth Model)
- Multiple of BV – multiply the ending total capital (in Terminal year) times some representative MV / BV multiple (current multiple for the company may be a good approximation unless expected to change over time)
- Multiple of earnings - multiply earnings (in Terminal year) times some representative P/E ratio (current multiple for the company may be a good approximation unless expected to change over time)
- liquidation value - calculate liquidation value in Terminal year; liquidation value might be some % of BV

II. Cash Flow Issues

- EBIT
  - \( EBIT = CFD + CFE + \text{taxes} \)
- Capital Cash Flows – Value of entire firm → CF to both D & E (including tax deductible interest)
  - \( CCF \) = after tax cash flows available to all security holder (D & E)
  - \( CCF \)
    - = Operating CF – tax
    - = ECF + DCF
    - = FCF + ITS
- \( OCF = \text{CF from operations + CF from Investments excluding CF to Capital holders (i.e., interest)} \)
- discount rate ~pre-tax \( r_A \) (i.e., expected return on assets)
  - \( r_A = r_F + (B_{\text{Asset}}) \) (risk premium)
- Equity Cash Flows – Value of Equity → CF available to S/H after payments to D/H
  - Debt Cash Flows ~ payments of interest and principal to D/H
  - valued as a perpetuity using \( r_E \)
  - \( ECF \)
    - = CCF – DCF
    - = OCF – tax – int – debt repay + debt proceeds issued
    - = (EBIT-r_D)(1-T) + Dep – ∆NWC – CAPX – ∆D
discount rate - \( r_E \) (~higher risk b/c subordinate to DCF \( \Rightarrow \) higher rate than \( r_A \))

- \( r_E = r_F + B_{Equity} \) (risk premium)
- \( B_{Leveled\ Equity} = B_{Asset} / (E/V) = B_{Asset} \cdot (V/E) \)

- Debt Cash Flows -
  - \( DCF = r_D D + \Delta D \)

- Free Cash Flows – Value of entire firm \( \Rightarrow \) value whole firm (but deals with taxes outside of CF by including in discount rate)
  - \( FCF = OCF - \text{tax} \)
  - valued as a perpetuity using WACC
  - discount rate \( \sim \) WACC = \( r_E (E/V) + r_D (D/V) \cdot (1-t) \)
  - note \( - r_A = \text{WACC} + \text{ITS} = \text{WACC} + t \cdot (D/V) \cdot r_D \)

- Summary –

\[
\begin{array}{|c|c|c|c|}
\hline
\text{CF} & \text{Discount Rate} & \text{Value} & \text{Comment} \\
\hline
\text{UFCF} & r_A & V_U & \text{special case where } D = 0 (~ D/V = 0) \\
\hline
\text{UFCF} & \text{WACC} & V_L & \text{special case where } D/V \text{ is fixed} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{CFD} & r_D & D & \text{NPV} = E - \text{equity investment (?)} \\
\hline
\text{CFE} & r_E & E & \text{NPV} = E - \text{equity investment (??)} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{If } D/V = \text{constant} \Rightarrow DCF & \text{Value} & \text{Comment} \\
\hline
V_L = V_U + DVITS & \text{NPV} = E - \text{equity investment (??)} & \text{special case where } D/V \text{ is fixed} \\
& \text{NPV}_U = V_U - C_O & \text{NPV}_L = V_L - C_O \\
& \text{DVITS} = TV_L - TV_U & \text{DVTS} = (1 \times T) / (1 + r_D) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{If } D/V \neq \text{constant} \Rightarrow APV & \text{Value} & \text{Comment} \\
\hline
V_L = V_U + DVITS & \text{NPV} = E - \text{equity investment (??)} & \text{special case where } D/V \text{ is fixed} \\
& \text{NPV}_U = V_U - C_O & \text{NPV}_L = V_L - C_O \\
& \text{DVITS} = TV_L - TV_U & \text{DVTS} = (1 \times T) / (1 + r_D) \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Return on Equity Method} & \text{Comment} \\
\hline
\sim \text{simply find IRR on CFE (if above hurdle rate } \Rightarrow \text{ accept)} & \text{BUT if } D/V \text{ is not constant } \Rightarrow \text{ no IRR exists} \\
& \text{BUT if NPV is positive at hurdle rate } \Rightarrow \text{ IRR m/b > hurdle rate} & \text{Decision tree} \\
& \text{better to use this method when default is imminent} & \text{~ each period } \Rightarrow \text{ calc probability will default next period} \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{IRR} & \text{Comment} \\
\hline
& \text{does not adjust for size} & \text{poor for mutually exclusive projects} \\
& \text{IRR}_{UFCF} \geq r_{WACC} & \text{IRR}_{CFF} \geq r_E \\
\hline
\end{array}
\]

III. Discount Rate Estimation

- CAPM
  - note
    - use b/c there is no better alt
    - estimate for single stock is very noisy
CAPM applies to any asset
- discount rate of any CF: \( k_i = r_f + B[E(r_m)-r_f] \)
- return on asset: sum of all cash flows + change in MV
- \( B \): measures magnification or attenuation of market moves on the value of asset i
  - measures proportionality of change in asset value relative to market change
- \( R^2 \): measures tightness with which market changes affect changes in asset value
  - measures fraction of change in asset value explained by market
- \( r_f \): return on a portfolio that guarantees no variation in wealth
- market: all possible investment alts
  - caveats:
    - most 3'th level worrying is unnecessary due to other sources of errors in process +
    - accurate benchmark: impossible & unnecessary SLA actual index is exposed to all major factors generally affecting the economy
  - index used:
    - in practice = S&P 500 (but is unrepresentative and skewed to US large caps)
    - better = Wilshire 5000 or Morgan Stanley World Index
  - implicitly relies on arbitrage theory — if asset is overpriced, investors will leave until P becomes better aligned
- discount rate:
  - required return needed to induce investors to take a position in an asset
  - expected return of an investor from investing in an asset

estimating \( B \)
- not observable BUT may be found thru regression
- typically — monthly returns over 5 yr period (trade-off b/w noise in ST and more observations over LT)
  - key: can use other firms to estimate \( B \)
- S/E affects reliability of coefficient estimate (generally not reported)
- reject an estimate either:
  - \( B \) estimate is noisy (~ if change horizon, \( B \) estimate changes substantially)
  - \( SE \) is so large as to make \( B \) unusable (i.e., 2 \( SE \) from \( \mu \))
    - BUT — \( SE \) of average of \( n \) estimates is divided by \( n^{0.5} \) iff \( average \) of \( n \) estimates of the same true \( B \)

estimating the Estimated Market Risk Premium (EMRP)
- upward bias in the security return if security has typical exposure to systematic risk (\( B = 1 \))
- main issues:
  - more observations v. timely data: we want more observations (not timeliness b/c we must drive down SE + can’t use other firms to estimate MRP (b/c there is only one market))
  - how compute avg MRP:
    - typically — S&P 500 from 1926 to present (but avg varies with period chosen)
    - problem: lots of noise (impossible to tell if EMRP varies over time) + can’t say when market s/b hi / low b/c no external references
    - avg: arithmetic mean (not geometric) b/c it better approximates the true (one yr) \( \mu \) return (due to Law of Large Numbers)
  - how to define \( r_f \): reference is US treasury over the period of the CF (despite lots of variation (annual return \( \sigma \)) with LT bonds)

CAPM cons
- a) no empirical relationship b/w \( B \) & return +
- b) other factors do better +
- c) difficult to justify projecting discount rate from many stocks onto one
- d) typical variability of \( B \) is large (and unknown)

Multi-factor models — SKIPPED

remember
- for illiquid assets \( \rightarrow \) MV may be stale (and return may be harder to calculate)
- CAPM ignores \( R^2 \) b/c diversification eliminates the fraction of change in asset value not determined by market changes
- CAPM is less a theory of expected returns and more of a theory of expected return premiums
- P multiple better than DCF at tracking trends in EMRP
- CAPM is theoretically graceful but not empirically sound
- ALWAYS consider stat variability of a B estimate in addition to its estimated coefficient

IV. Valuation Methods

- APV -
  - i) Value firm as if all equity financed (i.e., discount at $r_A$)
  - ii) Value ITS
    - discount rate should equal the riskiness of CF being valued
    - real risk is ~ that EBIT < interest
    - i.e., there is less risk of this than risk associated w/ EBIT itself (if firm is prudently leveraged)
    - thus – OK to use lower discount rate than that on equity
    - common convention ~ use pre-tax interest rate
  - iii) Value other "side effects associated w/ financing –
    - e.g., Tax loss carryforwards
    - discount at rate that reflects risk (~ pre-tax rate on debt)

- Equity Cash Flow Valuation –
  - in general
    - use for - valuing highly levered equity claims on operating assets
    - alternative to option pricing (which is preferable but may be impractical)
    - bene – while it gives a bias result (similar to other methods), at least we know it underestimates Equity Value
  - theory – owners of highly leveraged equity have call on asset
    - challenge ~ complex capital structure means that equity holders have sequence of nested options to exercise
      call any time one of myriad of debt obligations comes due
    - w/ lower leverage → less important to value owners option to walk away from debt
  - calculation -
    - $EFC = CCF - DCF = OCF - tax - (int - debt repay + debt proceeds issued)$ [see above]

V. Optimal Capital Structure

- Generally – interest deductibility means that firm may realize lower $R_A$ and still meet its required $R_E$ and $R_D$

- But –
  - A) Agency costs of debt – CEO may pursue C/F negative projects
  - B) Default risk –
    - calc value at given risk level and find probability of not being able to meet interest commitments
    - direct costs – legal / accy fees + lack of secondary market for assets
    - indirect costs – customers don’t want to deal (stigma) + COI b/w B/H & S/H + lack of control in bankruptcy
  - C) limited $\tau$ tax $\rightarrow$ means finite beneficial debt load
  - D) debt covenants likely will impede operations (especially as Debt load grows) $\sim$ limiting value transfers
  - E) riskiness of business $\rightarrow$ higher earnings volatility increases probability of insufficient int coverage
  - F) asset type – some assets are less liquid / have lower liquidation value than others (further impairing ability to cover interest in a pinch)

VI. Capital Budgeting

- measure of worth
  - payback period -
    - essentially ~ time for stream of CF to equal original cash outlay
    - cons – ignores post payback CF + ignores TVM
- **Avg RoR** -
  - essentially - proceeds divided by yrs during which received + then divide by original investment
  - cons – ignores TVM + ignores duration of proceeds (~ bias towards ST investments)

- **PV** – essentially – today’s value of money t/b received in future

- **NPV** –
  - process – choose discount rate + calc PV of cash receipts + calc PV of cash outflows + compare both PVs

- **IRR** –
  - essentially – find int rate that makes PV of cash inflows & outflows equal (~ T&E)
  - used to calculated yield to maturity of investment in gov’t bonds
  - if projects = mutually exclusive → use NPV (or choose one with higher IRR if NPV not available, but beware)
  - cons – ignores total return (~ mutually exclusive projects) + multi switch b/w in/out flows = multi IRRs + possible to have no IRR (if + - +) + fails if OCoC changes over time
  - pros – allows to eval projects w/o computing CoC

### Two Key issues

1) **C/F**
   - *include*
     - i) only relevant cash flows – incremental (~ OH) + after tax
     - ii) all incidental effects (erosion / other benes)
     - iii) working capital needs (~ WC includes cash needed to operate company
     - iv) sustaining Capx
     - v) opportunity cost – e.g., need to invest in supporting PPE early
     - vi) real option value
       - at least recognize effect (positive / negative) on alts
   - *exclude*
     - i) sunk costs
     - ii) allocated OH costs
     - iii) CF of unrelated projects

2) **discount rate** –
   - represents OCoC + reflects risk & TVM

### consider

- commodity sales → P should go down w/ more production
- sunk engineering costs → consider timing of larger project (concern = front loading)
- terminal value – may make up significant portion of value
  - value perpetuity (but recognize Capx to maintain)

### VII. Valuing an Acquisition Candidate (Kennecott)

- **why acquire**
  - **diversification** –
    - operating synergy → changing C/F
    - financial synergy → changing r
  - S/H can do this BUT
    - a) incomplete capital markets may make conglomerate more efficient (~ emerging markets)
    - b) size of conglomerate → gives ability to tap markets
    - mgmt talent may be scarce

- **Issues**
  - i) identify appropriate CF – any synergies?
– ignore Acquirer’s tax loss – this has nothing to do with any particular Target

- ii) what discount rate –
  - have to look at Target and its business risk
  - also depends on which CF you are discounting

- iii) what capital structure

VIII. Option Theory

- Defn
  - call – right to buy stock → all upside & total downside protection
  - put – right to sell stock → protection from P rise
  - premium – price paid by buyer to acquire option
  - exercise P – strike P – price paid to acquire underlying security
  - expiration date – last date option c/b exercised
  - European Option – exercised only at time of expiration
  - American Option – exercised at any time prior to / at expiration
  - Expected Return \( (r_{tm}) = (P_t - P_{t-1} + D_t) / P_{t-1} \)
  - geometric mean –
    - \( P_t = P_0 (1+r)^t \)
    - continuously compounded \( P_t = P_0 e^{rt} \)
      - \( r_t = \ln (P_t / P_{t-1}) \)
  - trading days – assume 252 in US

- Calculation
  - \( C \) – exercise P
  - \( P_{S,T} \) – stock P at time to expiration (T)
  - \( E \) - 2,7183
  - \( N(d_1), N(d_2) \) – value of cumulative normal distribution at \( d_1 \) and \( d_2 \)

  - Payoff Matrix -

<table>
<thead>
<tr>
<th></th>
<th>( P_S &lt; C )</th>
<th>( P_S &gt; C )</th>
<th>In other Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call Option (Long)</td>
<td>0</td>
<td>( P_S - C )</td>
<td>( \max (0, P_S - C) )</td>
</tr>
<tr>
<td>Put Option (Long)</td>
<td>( (C - P_S) )</td>
<td>0</td>
<td>( \max (0, C - P_S) )</td>
</tr>
</tbody>
</table>

- Put-Call Parity – construct portfolio to create synthetic “riskless” security

  - e.g.,
    - A) construct portfolio
      - i) purchase share (long in stock)
      - ii) buy a put option (long)
      - iii) write a call option (short)

    - B) calculate payoffs –

<table>
<thead>
<tr>
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<th>( P_S &gt; C )</th>
<th>In other Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Call Option (Short)</td>
<td>0</td>
<td>( - (P_S - C) )</td>
<td>( \max (0, P_S - C) )</td>
</tr>
<tr>
<td>ii) Put Option (Long)</td>
<td>( (C - P_S) )</td>
<td>0</td>
<td>( \max (0, C - P_S) )</td>
</tr>
<tr>
<td>iii) Stock</td>
<td>( P_S )</td>
<td>( P_S )</td>
<td>( P_S )</td>
</tr>
</tbody>
</table>

| Net Position     | \( P_{S,T} + P_{P,T} + P_{C,T} \) | \( + C \) | \( + C \) | \( + C \) |

- S/H v. B/H – equity writes a call option (short) to D/H

<table>
<thead>
<tr>
<th></th>
<th>( V_L &lt; D )</th>
<th>( V_L &gt; D )</th>
<th>In other Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>B/H</td>
<td>( V_L )</td>
<td>D</td>
<td>( \max (V_L, D) )</td>
</tr>
<tr>
<td>S/H</td>
<td>0</td>
<td>( V_L - D )</td>
<td>( \max (0, V_L - D) )</td>
</tr>
</tbody>
</table>
Binomial option pricing model – determining Option Premium by forcing ROI = rf

- says – if we know rf → we can determine Call Premium (since (i) can construct riskless security through put / call parity + (ii) force ROI to equal rf)

how
- i) identify possible states of the world
- ii) find combination of securities (put, call, stock) that create risk free return (i.e., return the same amount regardless of the state of the world that finally occurs)
- iii) determine value of securities in various states of world
- iv) determine P-option
  - a) rf = [Risk free payoff / (PS,0 − PCall)] − 1
  - b) P_option = PS,0 − [Risk free payoff / (1+rf)]

how - i) identify possible states of the world
- ii) find combination of securities (put, call, stock) that create risk free return (i.e., return the same amount regardless of the state of the world that finally occurs)
- iii) determine value of securities in various states of world
- iv) determine P-option
  - a) rf = [Risk free payoff / (PS,0 − PCall)] − 1
  - b) P_option = PS,0 − [Risk free payoff / (1+rf)]

e.g.,

<table>
<thead>
<tr>
<th>Value of Stock</th>
<th>Depression</th>
<th>Prosperity</th>
<th>In other Words</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS,D</td>
<td>PS,D</td>
<td>PS,D − (PS,P − C)</td>
<td>will not be same</td>
</tr>
<tr>
<td>Value of Option</td>
<td>0</td>
<td>- (PS,P − C)</td>
<td>focus on payoff to buyer</td>
</tr>
<tr>
<td>Total Payoff</td>
<td>PS,D</td>
<td>PS,D − (PS,P − C)</td>
<td>s/b same value</td>
</tr>
</tbody>
</table>

- solve for gamma – choose δ such that PS,D = PS,D − δ (PS,P − C)
  - δ = [PS,D - PS,P / (PS,P − C)] ~ ratio of range of PS to range of option payoffs

- solve for option P
  - ~ PCall = 1/δ [PS,0 − [PS,D / (1+rf)]] (~ where PS,D = risk free payout)

- European Call (1 period binomial)
  - priced as –
    - a) fully hedged long position in stock +
    - b) writing δ call options w/strike P of C
  - valuation =
    - i) PCall = 1/δ [PS,0 − [PS,D / (1+rf)]]
    - ii) 1/δ (PS,0) − (PS,D / δ) C) [C / (1+rf)]
    - iii) binomial distribution = HR1 (PS,0) − HR2 [C / (1+rf)]
      > where HR1 = 1 / δ and HR2 = (PS,D / δ C)

- Black Scholes Option Pricing Model

a) generally –
- assumptions – continuous trading in stocks & options + continuously compounded constant rf + σ of stock return is constant + no dividends + continuous RoR of stocks are normally distributed (~ stock P = log normally distrib) + P moves as random walk
- ignores – Dividends + American Options

- European Call -
  - PCall = N(d1) PS,0 - N(d2) (PS,0 e−rfT)C
  - PCall = HR1 PS,0 − HR2 (PS,0 e−rfT)C ~ similar to binomial distribution
  - where
    > d1 = [ln (PS,0 / C) + (rf + (σ²/2)) T] / σ(T0.5)
    > d2 = d1 − σ(T0.5) (note – assumes constant σ, but may not be)
implied volatility – compensates for fact that σ is not observable → found w/ iterative approach (no closed form inversion of problem) by - either
- a) estimate from time series data of stock returns
- b) estimate volatility by option premiums observed in market
- c) pure plays

variables in Option Pricing

<table>
<thead>
<tr>
<th>Traditional Option</th>
<th>Observable?</th>
<th>Capital Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) C - Strike Price</td>
<td>Y</td>
<td>Amt expended (X)</td>
</tr>
<tr>
<td>ii) σ² - Risk of underlying asset</td>
<td>N</td>
<td>σ² of project returns</td>
</tr>
<tr>
<td>iii) rF</td>
<td>Y</td>
<td>rF</td>
</tr>
<tr>
<td>iv) P,0 - Current P of underlying asset</td>
<td>Y</td>
<td>Value of asset built (S)</td>
</tr>
<tr>
<td>v) t – time to maturity</td>
<td>Y</td>
<td>time can wait w/o losing opp</td>
</tr>
</tbody>
</table>

b) Dividends
- American Call (no div) → no incentive to exercise before maturity (i.e., same as European)
- American Put (w or w/o div) → may be incentive to exercise when put is deep in money
- American Call (small Div) → not enough to early exercise → adjust BS to reflect that not entitled to div
  - $P^*_{S,0} = P^*_{S,0} - \text{Sum } e^{-rFt}D_t$
- American Call (large Div) → may be exercised early → see special valuation models if div are known
- Uncertain Div → not possible to form risk free hedge portfolios → Call options cannot be valued w/o considering risk preferences

c) Warrants
- note – dilution effect, but firm gets cash
- calculation -
  - N = O/S shares
  - M = O/S European warrants (one sh per warrant (t) yrs later at price (C))
  - $W = P_{\text{warrant}} = P_{\text{Call}} * N / (M+N) – accounts for dilution$
  - $P^*_{S,0} = P_{S,0} + (M/N)W$
  - i) identify N, M, t, σ, rF, C, P,0 (known or estimated)
  - ii) guess at W
  - [e.g., $W_t = N / (M+N) * P_{\text{Call}} (~ of equivalent warrant)]$
  - iii) calc N(d1) & N(d2)
  - iv) $P_{\text{Call}} = P^*_{S,0} * N(d_1) - N(d_2) Ce^{-rFT}$
  - vi) $W_E = N / (N+M) * P_{\text{Call}}$
  - vii) go to step (ii) until $W_B = W_E$

Real Options

* Background
  - consider –
    - downsizing (a) is investing against future losses + (b) closes off future opps (forfeits option to resume ops)
    - opportunity to invest in project is like a call option (investing kills the option’s value)
    - in practice → people discount at rates > OCoC
    - substantial value of most cos = options to invest / grow
    - investment = irreversible when
      - specific to industry (everyone sees as worthless) or company (~sunk costs)
      - gov’t regulation
      - differences in corp culture
    - option value = EV w/ option – EV w/o option
    - generally better to wait for uncertainty to be resolved
    - acts that create options → more valuable than NPV indicates
    - scale v. flexibility – latter may be nmore valuable in face of uncertainty
    - where large sunk costs exists → closing = opportunity cost b/c lose intangibles
  - cons of NPV =
    - Assumptions → investment is irreversible + now or never proposition + ignores value in creating options through current investment + in practice, WACC is simply used as disc rate
leads to missing valuable ops + missing the timing
  key question = when to exercise option

- option theory of investing – as concept progresses toward implementation → options disappear and value falls (hurdle rate rises)

- Capital Projects
  - valuation methods -
    - a) DCF – applies if project cannot be delayed (t = 0) or has no variance (σ = 0)
      > NPVa = PV (CF) / PV (Capex) → RoT ~ invest if greater than 1
    - b) option – when project c/b delayed
      > NPVq – still matters, but is influenced by riskiness of project
      > option value = look up % X underlying Asset Value (S) for NPVq and cumulative variance (σ (t)^0.5)

    - cumulative variance = σ^2 X t (~ variance of project times time remaining on project)
      measures how much things could change before time runs out

    RoT - capital project s/b pursued if PV(CF) > PV (Capex) at maturity of option

<table>
<thead>
<tr>
<th>IF</th>
<th>NPV</th>
<th>NPVq</th>
<th>(σ^2 t)^0.5</th>
<th>then</th>
</tr>
</thead>
<tbody>
<tr>
<td>i)</td>
<td>&lt; 0</td>
<td>&lt;1</td>
<td>zero</td>
<td>never exercise</td>
</tr>
<tr>
<td>ii)</td>
<td>&lt; 0</td>
<td>&lt;1</td>
<td>low</td>
<td>doubtful prospects</td>
</tr>
<tr>
<td>iii)</td>
<td>&lt; 0</td>
<td>&lt;1</td>
<td>med/high</td>
<td>less promising – require active development</td>
</tr>
<tr>
<td>iv)</td>
<td>&lt; 0</td>
<td>&gt; 1</td>
<td>high</td>
<td>very promising b/c high variance</td>
</tr>
<tr>
<td>v)</td>
<td>&gt; 0</td>
<td>&gt; 1</td>
<td>low/medium</td>
<td>wait if possible</td>
</tr>
<tr>
<td>vi)</td>
<td>&gt; 0</td>
<td>&gt; 1</td>
<td>zero</td>
<td>exercise now – will not change</td>
</tr>
</tbody>
</table>

- consider
  - we can value European Call w/ only cumulative variance and NPVq
  - option w/ σ or t = 0 have no variance and c/b valued w/ DCF
  - simplify complex projects ~ sequence of serially dependent projects
    > i) identify main uncertainty
    > ii) find upper / lower bounds
  - estimating σ
    > i) guess – systematic B and total risk are positively correlated in large samples of operating assets
    > ii) estimate – historical data / implied volatility from quoted option P (note – equity returns are levered and more volatile than underlying asset returns)
    > iii) simulate σ - Monte Carlo
    > iv) locate project w/i “tomato garden” plot – if know whether cumulative variance is high / low

- remember
  - lack of symmetry b/c can’t go to negative infinity
  - P0 ~ C (1 / 1+rf) t ~ Ce^{-rt} (if continuous compounding)
  - E(r) return on fully hedged position = rf → [Ps,D / (Ps,0 - δ P Call)] -1 = rf
  - Loan Guarantee = Risky Bond + Put on loan
    - PPut = F (V0, rf, T, D, σV)
      - where – σV^2 = Var (D + E) = Var(D) + Var(E) + 2Cov (D+E)
  - must adjust NPV for opportunity cost of option
  - exercising options kills option value
IX. Multiples Valuation

- **Process**
  1. Choose comparable firms
     - Note: stringent criteria may = too few firms
     - Exclude abnormal firms
     - Criteria: types of goods produced / technology implemented / clientele / size (units produced) / leverage
  2. Choose bases for multiples → consider industry relevant criteria
     - If can’t adjust for capitalization differences → value whole firm v. Operating Income
  3. Average across industry
  4. Project bases for valued firm

- **Multiples**
  1. \( \frac{P}{E} = \frac{P}{E} \times EPS \)
     - Trailing earnings → must be discounted one period when applied to future earnings (t+1 is when future earnings become trailing earnings)
     - Leading earnings → value future value of firm
     - \( \frac{P_0}{EPS_0} = \frac{\text{Div Payout ratio} \times (1+g)}{r - g} \)
     - \( g = \text{retention ratio} \times \text{ROE} = (1-\text{Div payout ratio}) \times \text{ROE} \)
  2. **Terminal Values**
     - Con ~ assumes past holds for future
  3. \( \frac{P}{Sales} \)
     - Con ~ may assume too much similarity (lose differentiation further down in F/S)
  4. \( \frac{F/A}{A/mple} \)
     - Key = how is relative size measured in industry? (~ gross F/A or net F/A)

- **Remember**
  1. Applying historic multiple to next year ignores future prospects
  2. **Neg earnings** → apply multiple to first positive yr and discount back to today
     - Can’t exclude negative earnings b/c introduces downward bias
     - Solution ~ Sum Industry Values / Sum Industry earnings
  3. **Cons** → earnings manipulated by accts
X. Portfolio Risk

- generally –
  - is NOT wghted avg of σ of stocks in portfolio (unless stocks are perfectly correlated)
  - calculated by
  - i) identify % in each portfolio ($x_1$ & $x_2$)
  - ii) find σ of individual stocks
  - iii) find covariance of individual stocks
    - covariance = either
      - a) correlation coefficient $\sigma_{12} X \sigma_1 \sigma_2 X x_1x_2$
      - b) $\sigma_{12} = \text{expected value of } (\text{actual return}_1 - \text{exp return}_1) \times (\text{act return}_2 - \text{exp return}_2)$
  - iv) weight covariance by proportion of holdings ($x_1$ & $x_2$)
  - v) multiply squared proportion of holding X variance (for $x_1$ & $x_2$ separately)
  - vi) portfolio variance = $x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2(\text{correlation coefficient}_{12} \sigma_1 \sigma_2 x_1x_2)$

- matrix -

<table>
<thead>
<tr>
<th></th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>Stock 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock 1</td>
<td>$x_1^2 \sigma_1^2$</td>
<td>$x_1x_2 \sigma_{12} = x_1x_2 \rho_{12} \sigma_1 \sigma_2$</td>
<td>$x_1x_3 \sigma_{13} = x_1x_3 \rho_{13} \sigma_1 \sigma_3$</td>
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<tr>
<td>Stock 2</td>
<td>$x_2x_1 \sigma_{21} = x_2x_1 \rho_{21} \sigma_2 \sigma_1$</td>
<td>$x_2^2 \sigma_2^2$</td>
<td>$x_2x_3 \sigma_{23} = x_2x_3 \rho_{23} \sigma_2 \sigma_3$</td>
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<tr>
<td>Stock 3</td>
<td>$x_3x_1 \sigma_{31} = x_3x_1 \rho_{31} \sigma_3 \sigma_1$</td>
<td>$x_3x_2 \sigma_{32} = x_3x_2 \rho_{32} \sigma_3 \sigma_2$</td>
<td>$x_3^2 \sigma_3^2$</td>
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- Portfolio Variance = sum of all boxes

#### Variance terms

#### Covariance terms